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AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

No. 866.

COMPARISON OF WEIGHTS OF A THREE-HINGED AND A TWO-HINGED SPANDREL-BRACED PARABOLIC ARCH.

BY C. W. HUDSON, ASSOC. M. AM. SOC. C. E.

PRESENTED SEPTEMBER 20TH, 1899.

WITH DISCUSSION.

In June, 1896, the author had occasion to calculate the stresses, sections and weight of a three-hinged spandrel-braced parabolic arch, the outline and main dimensions of which are shown in Fig. 1. Shortly after finishing these calculations he determined to make the corresponding calculations for an arch of the same outline having two hinges, in order to determine their exact relative economy as regards weight of metal. It was not until recently, however, that the second calculation was completed. As it would seem that these results might be of some interest to members of this Society, they are here given, together with a brief exposition of the method used in calculating the stresses in the two-hinged arch.

These arches were designed for a live load of 2 160 lbs. per lineal foot or 36 000 lbs. per panel of the arch, and a dead load of 2 880 lbs. per lineal foot or 48 000 lbs. per panel of the arch. This unusually large dead load is due to a heavy asphalt floor carried by buckle plates.

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No metal less than $\frac{5}{16}$ in. in thickness was used for either case; and only two sizes of channels—12 and 15 ins.—were used for the members other than the arch ring. By introducing another shape, say 10-in. channels, a small saving of weight could have been made in both cases, but the appearance of the arch is better on account of the greater uniformity, and the construction is cheapened for the same reason.

SPANDREL-BRACED PARABOLIC ARCH

CASE I.—WITH THREE HINGES.

CASE II.—WITH TWO HINGES.

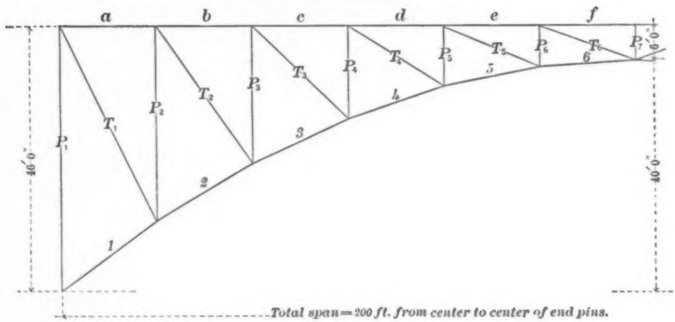

FIG. 1.

The unit stresses used in proportioning the members were:

For tension

Live-load stresses, 11 000 lbs. per square inch.

Dead-load " 22 000 " "

For compression

In arch ring and top chord

Live-load stresses, $12\,000 - 55\frac{l}{r}$ lbs. per square inch.

Dead-load " $24\,000 - 110\frac{l}{r}$ " "

In web members

Live-load stresses, $11\,000 - 50\frac{l}{r}$ lbs. per square inch.

Dead-load " $22\,000 - 100\frac{l}{r}$ " "

It would have been more consistent and better to have used for the arch rings and top chords also the compression formulas used for

web members, as the loading producing maximum stress in these members is partial, not covering the entire span, for almost all the members of both cases.

These unit stresses, as will be recognized, are those for medium steel, of Theodore Cooper's Specifications of 1896 for Highway Bridges.

The temperature stresses in the two-hinged arch were treated as dead-load stresses. Members subject to alternate stresses of tension and compression were proportioned to resist both kinds of stress, with $\frac{8}{10}$ of the smaller added to either.

TABLE No. 1.—CASE I. STRESSES, SECTIONS AND WEIGHTS FOR A THREE-HINGED ARCH HAVING A SPAN LENGTH OF 200 FT. FROM CENTER TO CENTER OF END PINS.

Member.	Live-load stresses.	Dead-load stresses.	Sections.	Weight in pounds.
			Sq. ins.	
P_1	+ 59 000 / − 77 000	24 000	2 15″ channels 100 = 19.6	
P_2	+ 50 400 / − 86 400	48 000	2 12″ " 89 = 17.4	
P_3	+ 37 900 / − 73 900	48 000	2 12″ " 65 = 12.7	
P_4	+ 24 700 / − 60 700	48 000	2 12″ " 65 = 12.7	14 230
P_5	+ 11 700 / − 47 700	48 000	2 12″ " 65 = 12.7	
P_6	+ 30 600 / − 66 600	48 000	2 12″ " 65 = 12.7	
P_7	− 36 000	48 000	2 12″ " 65 = 12.7	
T_1	± 65 800	....	2 12″ " 100 = 19.6	
T_2	± 61 600	....	2 12″ " 80 = 15.7	
T_3	± 54 900	....	2 12″ " 65 = 12.7	
T_4	± 51 100	....	2 12″ " 65 = 12.7	15 560
T_5	± 49 600	....	2 12″ " 65 = 12.7	
T_6	± 130 800	....	2 12″ " 138 = 27.1	
a	± 29 200	....	2 15″ " 96 = 18.8	
b	± 64 500	....	2 15″ " 96 = 18.8	
c	± 103 100	....	2 15″ " 96 = 18.8	
d	± 134 300	....	2 15″ " 124 = 24.3	14 230
e	± 122 800	....	2 15″ " 124 = 24.3	
f	....	....	2 15″ " 96 = 18.8	
1	− 334 800	446 400	4 Ls. 4 × 4 × 48, 2 pls. 24 × $\frac{11}{16}$ = 51.8	
2	+ 7 100 / − 321 900	419 800	4 Ls. 4 × 4 × 44, 2 pls. 24 × $\frac{11}{16}$ = 50.3	
3	+ 21 600 / − 319 600	397 300	4 Ls. 4 × 4 × 48, 2 pls. 24 × $\frac{5}{8}$ = 48.8	
4	+ 41 500 / − 326 200	379 500	4 Ls. 4 × 4 × 44, 2 pls. 24 × $\frac{5}{8}$ = 47.3	36 180
5	+ 53 700 / − 329 200	367 100	4 Ls. 4 × 4 × 44, 2 pls. 24 × $\frac{5}{8}$ = 47.3	
6	+ 10 600 / − 281 200	360 800	4 Ls. 4 × 4 × 44, 2 pls. 24 × $\frac{9}{16}$ = 44.3	
			Details (from shop drawings)....	34 800
			Total weight =	115 000

Case II.—The stresses for the two-hinged arch are not statically determinate, but may be obtained from the elastic properties of the arch.

The vertical components of the reactions for any load are the same as the reactions for a simple truss of the same span. The horizontal component of the reactions for any load can be found from the following formula:*

$$H = P \frac{\delta}{\delta'}$$

In which

H = Horizontal thrust produced by P;

P = Load at any point;

δ = Vertical deflection of the loaded point, due to a horizontal force of unity acting at the free hinge—one hinge assumed fixed and the other free—;

δ' = Horizontal displacement of the free hinge, due to a horizontal force of unity acting at the free hinge.

Fig. 2 will make the notation of this formula more clear than the definitions alone could.

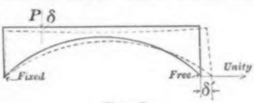

FIG. 2.

For simplicity of comparison the results are arranged as compactly as possible in Table No. 2.

The first column gives the marks designating the members (see Fig. 1). The second column shows the stresses due to a horizontal force of unity acting at one of the hinges. These stresses were very carefully figured analytically, and then checked graphically by means of the diagram, Fig. 3. From these stresses the changes in length λ of

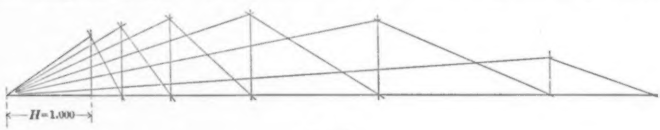

FIG. 3.

the various members were computed from the formula $\lambda = \frac{TL}{EA}$. In determining these values of λ, which are given in the third column, L was taken in feet, A, at present unknown, was taken equal to unity, and E was taken at 29 instead of 29 000 000, in order to give values of λ sufficiently large to allow them to be accurately plotted. These pre-

* "Roofs and Bridges," Part IV, Merriman and Jacoby.

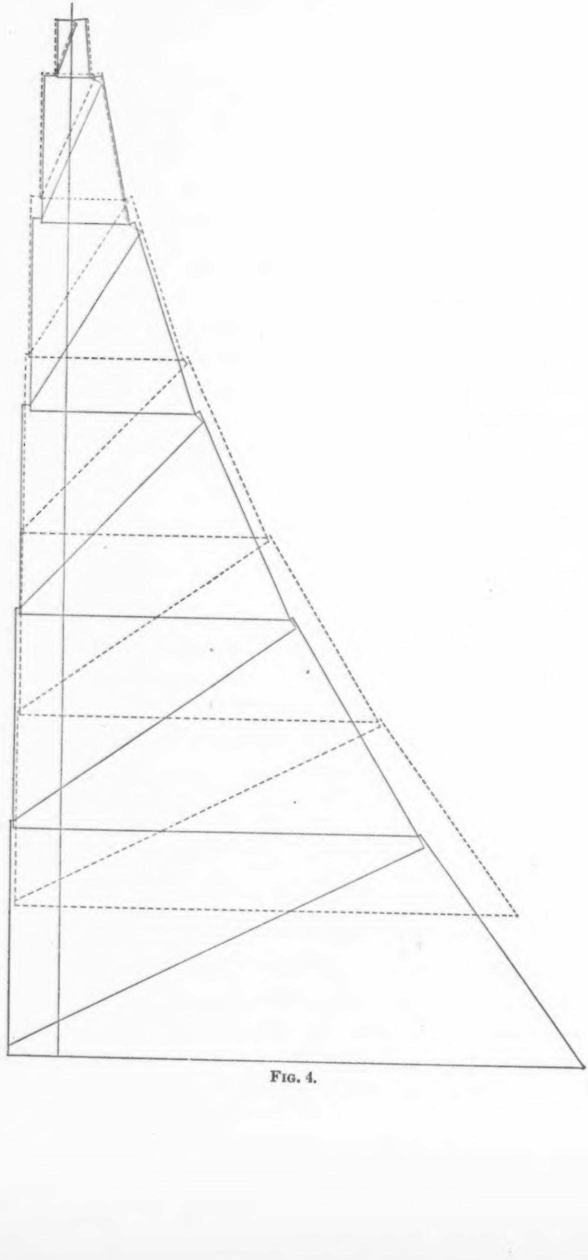

FIG. 4.

liminary values of λ are 1 000 000 $\times$ A times their true value. Using these values of λ the displacement diagram shown in dotted lines in Fig. 4 was constructed. The full lines in this figure give the final displacement diagram, and from this diagram the ratio of δ to δ' was found to have the following values, beginning with the panel point at the end of the arch: 0.003, 0.203, 0.401, 0.590, 0.764, 0.904 and 0.962.

The thrust due to a load at the end of the span being so small, only 0.003 of the load, it is neglected in both the preliminary and final calculations.

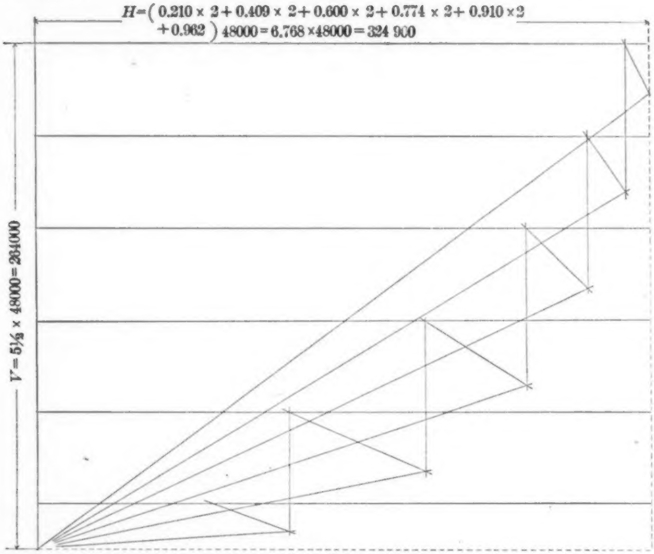

FIG. 5.

For full loading the preliminary value of the horizontal thrust is 6.686 $\times$ 48 000 lbs.; from this, by means of a diagram similar to Fig. 5, (Fig. 5 being the final dead-load stress diagram), the preliminary dead-load stresses are obtained. These stresses are given in the sixth column of the table.

Before finding the preliminary live-load stresses it will be well to find the stresses in the arch due to a vertical reaction of unity, supposed to be applied in this case at the left hinge. The stresses due to this load are given in the fifth column of the table. They were com-

TABLE No. 2.—(*Continued*).

a ..	+ 0.362	+ 0.208	+ 0.011	— 0.493	— 14 500	— 12 700	+ 20 100 — 30 700	+ 20 800 — 30 200	3 600	2 15″ " 96 = 18.8	2 15″ " 96 = 18.8	
b ..	+ 0.935	+ 0.537	+ 0.029	— 7.009 — 1.402	— 36 500	— 33 200	+ 41 400 — 68 700	+ 43 000 — 67 500	9 400	2 15″ " 96 = 18.8	2 15″ " 96 = 18.8	
c...	+ 1.875	+ 1.078	+ 0.057	— 9.375 — 3.125	— 74 500	— 66 300	+ 61 000 — 115 500	+ 64 000 — 113 400	18 800	2 15″ " 96 = 18.8	2 15″ " 96 = 18.8	
d...	+ 3.404	+ 1.956	+ 0.059	—12.766 — 6.383	— 134 100	— 120 300	+ 66 500 — 166 300	+ 71 500 — 166 900	34 000	2 15″ " 169 = 33.2	2 15″ " 159 = 31.2	
e...	+ 5.469	+ 3.143	+ 0.095	—16.406 — 11.719	— 214 700	— 192 000	+ 36 100 — 196 300	+ 44 200 — 188 300	55 000	2 15″ " 169 = 33.2	2 15″ " 159 = 31.2	
f...	+ 6.667	+ 3.831	+ 0.107	—16.667	— 260 900	— 232 800	— 195 400	+ 800 — 176 400	66 700	2 15″ " 183 = 35.9	2 15″ " 166 = 32.6	17 890
1...	— 1.240	— 0.884	— 0.018	+ 0.000	— 398 000	— 403 000	— 298 500	— 302 100	12 400	4 Ls. 4 × 4 × 51 2 pls. 20 × $\frac{3}{4}$ = 50.0	4 Ls. 4 × 4 × 53 2 pls. 20 × $\frac{3}{4}$ = 50.8	
2...	— 1.588	— 1.064	— 0.023	+ 0.575	— 357 600	— 364 000	+ 7 400 — 275 700	+ 7 000 — 281 200	15 900	4 Ls. 4 × 4 × 51 2 pls. 20 × $\frac{5}{8}$ = 45.0	4 Ls. 4 × 4 × 53 2 pls. 20 × $\frac{5}{8}$ = 45.8	
3...	— 2.135	— 1.354	— 0.033	+ 7.735 + 1.547	— 313 200	— 321 800	+ 23 200 — 258 600	+ 22 000 — 260 200	21 400	4 Ls. 4 × 4 × 41 2 pls. 20 × $\frac{5}{8}$ = 41.1	4 Ls. 4 × 4 × 43 2 pls. 20 × $\frac{5}{8}$ = 41.9	
4...	— 3.031	— 1.836	— 0.047	+ 9.882 + 3.294	— 259 900	— 272 900	+ 47 600 — 243 500	+ 44 900 — 249 700	30 300	4 Ls. 4 × 4 × 41 2 pls. 20 × $\frac{9}{16}$ = 38.6	4 Ls. 4 × 4 × 43 2 pls. 20 × $\frac{9}{16}$ = 39.4	
5...	— 4.492	— 2.633	— 0.083	+13.019 + 6.509	— 190 600	— 209 100	+ 74 000 — 217 000	+ 68 300 — 224 700	44 900	4 Ls. 4 × 4 × 29 2 pls. 20 × $\frac{1}{2}$ = 31.4	4 Ls. 4 × 4 × 29 2 pls. 20 × $\frac{9}{16}$ = 33.9	
6...	— 6.483	— 3.735	— 0.117	+16.442 +11.745	— 107 000	— 133 400	+ 72 000 — 152 500	+ 62 400 — 162 100	64 800	4 Ls. 4 × 4 × 29 2 pls. 20 × $\frac{1}{2}$ = 31.4	4 Ls. 4 × 4 × 29 2 pls. 20 × $\frac{1}{2}$ = 31.4	30 560
											Details (estimated)....	31 900
											Total weight =	109 000

*The Temperature Stresses in the Final Calculations were found to be the same as in the Preliminary Calculations.

TABLE No. 2.—CASE II.—STRESSES, SECTIONS, WEIGHTS, ETC., FOR A TWO-HINGED ARCH, HAVING A SPAN LENGTH OF 200 FT. FROM CENTER TO CENTER OF END PINS.

Truss Members.	Stresses due to H.	λ		Stresses due to V.	Dead-Load Stresses.		Live-Load Stresses.		Temperature Stresses.	Sections.		Weights of final sections in pounds.
		Prelim.	Final.		Prelim.	Final.	Prelim.	Final.	Preliminary and Final.*	Preliminary.	Final.	
										Sq. Ins.	Sq. Ins.	
P_1..	+ 0.733	+ 1.162	+ 0.062	− 1.000	− 53 000	− 49 800	+ 40 700 − 80 300	+ 42 000 − 79 400	± 7 300	2 15″ channels 96 = 18.8	2 15″ channels 96 = 18.8	
P_2..	+ 0.817	+ 0.952	+ 0.051	− 2.257 − 1.296	− 80 000	− 76 900	+ 30 800 − 90 800	+ 32 000 − 89 600	8 200	2 15″ " 96 = 18.8	2 15″ " 96 = 18.8	
P_3..	+ 0.903	+ 0.740	+ 0.044	− 2.271 − 1.654	− 85 200	− 79 300	+ 20 100 − 82 500	+ 21 400 − 81 100	9 000	2 12″ " 85 = 16.7	2 12″ " 82 = 16.1	
P_4..	+ 0.958	+ 0.529	+ 0.039	− 2.125 − 2.042	− 86 000	− 82 000	+ 8 500 − 72 700	+ 10 500 − 71 900	9 600	2 12″ " 70 = 13.7	2 12″ " 69 = 13.5	
P_5..	+ 0.881	+ 0.317	+ 0.025	− 1.553 − 2.277	− 82 700	− 79 000	+ 9 900 − 71 600	+ 10 700 − 69 900	8 800	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
P_6..	+ 0.431	+ 0.106	+ 0.008	− 0.094 − 1.781	− 64 600	− 62 800	+ 27 600 − 76 200	+ 28 100 − 75 500	4 300	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
P_7..	....	....	....	....	− 48 000	− 48 000	− 36 000	− 36 000	....	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	15 220
T_1..	− 0.818	− 1.062	− 0.084	+ 1.115	+ 32 200	+ 28 800	− 45 500 + 69 500	− 46 900 + 68 400	8 200	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
T_2..	− 0.998	− 0.999	− 0.079	+ 2.756 + 1.583	+ 38 900	+ 35 300	− 37 000 + 66 000	− 39 100 + 65 400	10 000	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
T_3..	− 1.304	− 1.039	− 0 082	+ 3.279 + 2.389	+ 53 000	+ 45 500	− 29 000 + 67 200	− 30 900 + 65 100	13 000	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
T_4..	− 1.805	− 1.224	− 0.097	+ 4.002 + 3.845	+ 70 900	+ 63 600	− 19 000 + 72 000	− 22 800 + 70 300	18 000	2 12″ " 65 = 12.7	2 12″ " 65 = 12.7	
T_5..	− 2.245	− 1.403	− 0.095	+ 3.958 + 5.801	+ 88 000	+ 78 300	− 39 400 + 105 600	− 42 400 + 101 400	22 500	2 12″ " 79 = 15.5	2 12″ " 75 = 14.7	
T_6..	− 1.273	− 0.778	− 0.033	+ 0.277 + 5.259	+ 49 300	+ 43 000	− 118 700 + 155 700	− 122 800 + 154 200	12 700	2 12″ " 120 = 23.5	2 12″ " 121 = 23.7	13 430

puted analytically and checked by means of the diagram, Fig. 6. The upper figures in the fifth column, where two sets are given, are the stresses for the corresponding member in the right half of the arch.

To determine the greatest tensile and compressive live-load stresses to which any member of the arch is subjected, take *5* of the arch ring for example, and arrange the computations as follows:

$$
\begin{array}{rl}
 & -H \\
4.492 & \left.\begin{array}{l}\times 0.203 \\ \times 0.401 \\ \times 0.590 \\ \times 0.764\end{array}\right\} 1.958 \quad (\times 0.764 = 3.432) \\
 & \left.\begin{array}{l}\times 0.904 \\ \times 0.962 \\ \times 0.904 \\ \times 0.764 \\ \times 0.590 \\ \times 0.401 \\ \times 0.203\end{array}\right\} 4.728 \quad (\times 0.904 = 4.061)
\end{array}
\qquad
\begin{array}{rl}
 & +V \\
13.019 & \left.\begin{array}{l}\times \frac{1}{12} \\ \times \frac{2}{12} \\ \times \frac{3}{12} \\ \times \frac{4}{12}\end{array}\right\} \tfrac{5}{6} = 10.849 \quad (\times \tfrac{4}{12} = 4.340) \\
6.509 & \left.\begin{array}{l}\times \frac{7}{12} \\ \times \frac{6}{12} \\ \times \frac{5}{12} \\ \times \frac{4}{12} \\ \times \frac{3}{12} \\ \times \frac{2}{12} \\ \times \frac{1}{12}\end{array}\right\} 2\tfrac{1}{3} = 15.208 \quad (\times \tfrac{7}{12} = 3.797)
\end{array}
$$

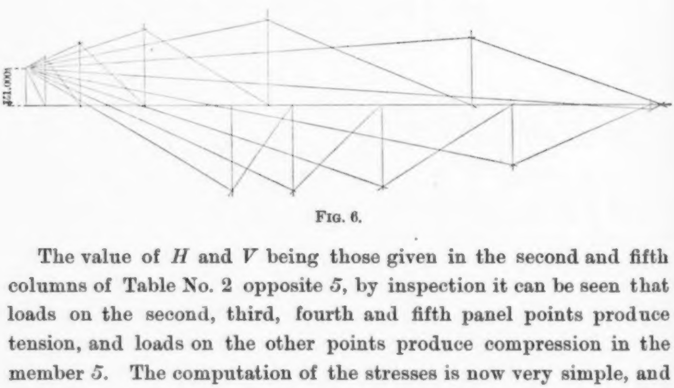

FIG. 6.

The value of *H* and *V* being those given in the second and fifth columns of Table No. 2 opposite *5*, by inspection it can be seen that loads on the second, third, fourth and fifth panel points produce tension, and loads on the other points produce compression in the member *5*. The computation of the stresses is now very simple, and we have:

For tension $(10.849 - 4.492 \times 1.958)\ 36\,000 = 2.054 \times 36\,000 = 74\,000.$

For compression $(4.728 \times 4.492 - 15.208)\ 36\,000 = 6.030 \times 36\,000 = 217\,000.$

The other live-load stresses were determined in this manner, and the computations were equally simple.

Having determined the preliminary live and dead-load stresses, it now remains to determine the preliminary temperature stresses. From the displacement diagram, already constructed, the horizontal displacement of the hinge, due to a horizontal force of unity, is found to be $\frac{230.8}{1\,000\,000 \times a}$ ft. It will be assumed that the arch will be subjected to a range of temperature of 120° Fahr., or a variation of 60° from the mean. This is a less variation than is usually assumed, but as the metal of the arch under consideration is to be protected from the direct rays of the sun by a highway floor, it is probably enough. The change in length of the arch due to a variation of 60° Fahr., taking the coefficient of expansion as 0.0000065 per degree Fahr., is 0.078 ft. Dividing this change of length due to temperature by the change in length due to a thrust of 1 lb., we get $\frac{78 \times a \times 10\,000\,000}{1\,000 \times 2\,308}$ lbs. as the value of the temperature thrust. The value of a in this expression for the value of the temperature thrust is at present unknown; the average influence of the areas of all the members of the arch is very nearly represented by the influence of the area of a member of the top chord near the center of the arch. In this case the approximate value of the area of the member e, as determined from the preliminary live and dead-load stresses, is used as the value of a in the expression for the temperature thrust. This approximate determination of the area of e gives about 31 sq. ins., which gives for the preliminary temperature thrust $\frac{78 \times 31 \times 10\,000\,000}{1\,000 \times 2\,308} = 10\,000$ lbs. The preliminary temperature stresses are now determined by multiplying the stresses in the second column of the table by 10 000.

From these preliminary dead-load, live-load and temperature stresses the preliminary sections were determined.

Using these new areas, instead of unity used in the preliminary calculation, the final calculations of stresses are made in exactly the same manner as the preliminary calculations. It is found that the preliminary temperature stresses need not be changed for the final determination of the sections. A final determination of the sections shows only small changes from the preliminary. In most of the members there is no change, and the greatest change amounts to only 6%, and as the preliminary determination for this case was on the safe side it seems that the final calculation was hardly necessary. Where

an estimate only is required, the preliminary calculation would certainly be ample.

The calculations of weight give:

For Case I.	115 000 lbs.
For Case II.	109 000 "

This gives a saving in weight of $5\frac{1}{2}\%$ in favor of the two-hinged arch. A variation of $75°$ in temperature would have lessened the difference considerably, but the two-hinged arch would still have been lighter.

While it is hardly allowable to draw general conclusions from the consideration of a special case, it can be said that where an arch of this form (spandrel-braced) is suitable, the two-hinged arch is lighter than the three-hinged arch; it is also cheaper to construct, as there is no center pin and there are no adjustable members at the center of the arch. The floor system of the two-hinged arch would also be more simple than that of the three-hinged arch, for the great range of height of the center-pin of the three-hinged arch, due to temperature and live-load stresses, makes a troublesome break in the floor system at this point.

When spandrel-braced arches are used in series, supported on intermediate masonry piers, the two-hinged arch has the advantage of having less horizontal thrust, and therefore requiring smaller piers than the three-hinged arch. Great care must be taken in the construction of the masonry for the two-hinged arch, in order that it may not be subject to even slight settlement or displacement; but, taking this extra work into account, it is believed that the masonry for a series of two-hinged arches will cost less than the masonry for a series of three-hinged arches.

DISCUSSION.

HENRY S. JACOBY, Assoc. Am. Soc. C. E. (by letter).—Since the adoption of the design of the spandrel-braced steel railway arch over the Niagara River, and the beginning of its construction, the writer has taken a renewed and more definite interest in the comparative study of different types of metallic arches. At his suggestion, two graduate students in the College of Civil Engineering of Cornell University last year made the computations necessary to determine the relative weight of three-hinged and two-hinged spandrel-braced arches having the same general dimensions and loading as the Niagara Railway Arch. Mr. George G. Smith, Jr., made the design for the two-hinged arch, while Charles C. More, Jun. Am. Soc. C. E., made the design for the three-hinged arch as well as that of a combination-type arch, which will be described in the latter part of this discussion. Mr. Jacoby.

The general dimensions, loading and specified unit stresses are given in the paper on "The Niagara Railway Arch."*

In finding the stresses in the trusses, however, two excess panel loads were substituted for the excess of the two locomotives above the corresponding weight of the train.

The Two-Hinged Arch.—In view of the statement made on page 137 of that paper, the given sectional areas of the members of the arch were not used except to find the stresses adopted in re-designing the sections. After this an additional revision was made, in order to see whether the final sections should generally depend upon the second or the third series of stresses, the first series being the preliminary ones which depend upon an assumption of equal sectional areas for all the members.

It was found that the required areas of the upper chord were increased from 0.5 to 1.1%, except those of U_7 and U_8 in Fig. 7, in which the increase was 2.9 and 3.3%, respectively. The areas of L_6 and L_7 were increased 0.5%, while those of the remaining lower-chord members were diminished from 0.4 to 1.3 per cent. The areas of the diagonals were increased from 0.2 to 1.8%, except that of D_5, in which the increase was 6.8 per cent. In the verticals, the increase of section varied from 1.2 to 3.5%, except

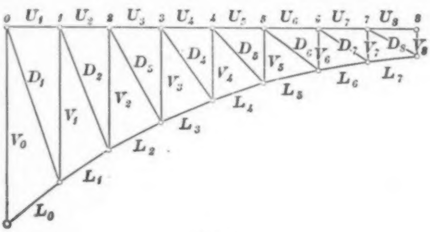

FIG. 7.

* By R. S. Buck, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xl, p. 125.

Mr. Jacoby. for V_7, in which it was 6.8 per cent. Since most of these differences were covered by the excess of the sections adopted over those required, only a few members were affected by this revision.

These changes in section are relatively much smaller than one is led to expect from the corresponding differences in the stresses, the result being due to the influence of the reverse stresses in the design. The temperature stresses were derived for a range extending from 75° above to 75° below the standard.

The above results indicate that the desirability of making the second revision depends largely upon the magnitude of the structure, while it is evidently affected to some extent by the form and proportion of the trusses.

The Three-Hinged Arch.—The dead-load stresses for the three-hinged arch were first found by using the same panel loads as those which had been determined from the design of the two-hinged arch. After this preliminary design was completed the dead panel loads were computed and found to exceed the previous values by percentages varying from 0.8 near the middle to 3.6 near the ends of the truss, while the middle panel load was 0.3% less. The average excess is 2.32 per cent. The dead-load stresses were revised accordingly, and the following differences were found, expressed as percentages of the sum of the dead, live and 0.8 times the reverse stresses. For the upper chord, 0.8 — 1.9; for the lower chord, 1.3 — 2.0; for the diagonals, 0.3 — 2.0; and for the verticals, 0.2 — 2.4. All the stresses were increased, except those in D_4, V_6, V_7 and V_8.

These changes, however, affected the sectional areas of only four members, the rest being covered by the small excesses in area made necessary by the make-up of the sections.

Comparative Weights.—Excluding the connecting plates, rivet heads and some minor details, all of which may be reasonably assumed to vary in the same ratio as the members of the respective trusses, it was found that the trusses of the three-hinged arch were 0.84% lighter than those of the two-hinged arch. The material is distributed as shown in Table No. 3.

TABLE No. 3.

	Upper chords.	Lower chords.	Diagonals.	Verticals.
Two-hinged arch	15.6%	36.7%	22.2%	25.5%
Three-hinged arch	12.9%	43.4%	20.1%	23.6%

It will be observed that in the three-hinged arch a considerably larger proportion of the material is contained in the lower chord, while the other classes of members contain less material than the two-hinged

arch. Only four members in these classes, U_3, U_4, D_1 and D_2 are larger in the three-hinged arch. It may be of interest to note that the range of stress in the lower chord of the three-hinged arch is 6.5% less, while the maximum stress (excluding the wind) averages 17.0% more, and the live-load stress 26.8% more, than in the two-hinged arch. Mr. Jacoby.

In order that the influence of the crown hinge might be obscured as little as possible by extraneous conditions, the designs of the corresponding members in the two-hinged and three-hinged arches were made as nearly alike in every respect as the required sectional areas would permit, the differences in the composition or make-up of the sections from that used in the construction of the Niagara Railway Arch, being reduced to a minimum. Since the distance from back to back of the angles in the chords was made the same in all cases, the additional material required in the lower chord of the three-hinged arch had generally to be placed on the inside and in the web plates, the result being to reduce the squares of the radii of gyration by an average of 4 per cent. This element places the three-hinged arch at a slight disadvantage.

Comparison with the Arch of 200-Ft. Span.—The corresponding distribution of material in the trusses described by the author is given in Table No. 4.

TABLE No. 4.

	Upper chord.	Lower chord.	Diagonals.	Verticals.
Two-hinged arch	23.2%	39.6%	17.4%	19.7%
Three-hinged arch	17.7%	45.1%	19.4%	17.7%

The web members are relatively lighter than in Table No. 3, while the chords are heavier, notably the upper chord. Some of this disparity may be accounted for by the difference in the working stresses, as well as the differences in the relative loading and in the proportions of the trusses. The uniform live load in the Niagara Arch is 2.3 times as great, while the dead load is only 1.8 times as great per linear foot as in this arch. Again, the span is 2.75 times as great, while the depth at the crown is 3.33 times as great. The ratio of the rise of the lower chord to the span is very nearly the same. The trusses of the Niagara Arch are about 10 times as heavy as those of the smaller arch. The range of temperature was assumed as 25% greater. The differences between the required sectional areas and those adopted in the design are relatively much larger in the smaller arch on account of the smaller sections, and the greater influence of the limitations imposed by the minimum thickness of metal allowed.

The different conditions just stated likewise affect the relative

Mr. Jacoby. weights of the trusses of the two types of arches, but it would require additional investigation to determine the magnitude of this effect.

One other element should be mentioned, namely, the effect of the wind stresses upon the truss members. For the comparative designs of the Niagara Arch it was assumed that no section should be increased in area unless the wind stress exceeded 40% of the sum of the dead, live and temperature stresses. It was found that this required an increase in only two chord members near the middle in the two-hinged arch, and one in the three-hinged arch.

It may, perhaps, be well to call attention to the fact that if in the smaller arch described in the paper the weight of the details be excluded, the difference in favor of the two-hinged arch is reduced to 3.9%, while if the details are included and made the same in both cases the differences will be only 2.7 per cent.

A Combination-Type Arch.—Any slight yielding in the foundations of a two-hinged arch materially changes the stresses, especially near the crown, while inaccuracies in the construction, in locating the end hinges, or in adjustment in closing the arch, have a similar effect. In view of these facts, it appeared to the writer that it would be desirable to erect an arch with three hinges, and, after completion, when all the dead load is in place, to transform it into a two-hinged arch by connecting, at the standard temperature, the upper chord at the center and by riveting the connection at the crown hinge. Such an arrangement eliminates the effect of any inaccuracies in construction or erection, as well as those of the initial set of the abutments, due to the imposed loads. The resulting stresses are a combination of the dead-load stresses of a three-hinged arch with the live-load and temperature stresses of a two-hinged arch.

It seemed worth while to determine how the weight of such an arch would compare with the other two types, and whether it would possess any other merit than that of realizing more perfectly in construction the conditions which are assumed in its design than is done by the two-hinged arch.

In the first determination of the sectional areas the stresses obtained in the preceding designs were used, and then a revision was made with the aid of a new set of live and temperature stresses. The temperature stresses were $10\frac{3}{4}$% greater than before. In this revision the required sectional areas were increased, with three exceptions, by from 0.4 to 8.2%, the difference being below 2% in all but a few members. One-half of the areas at first adopted were increased by amounts varying from 1.0 to 3.8%, except one which had to be enlarged 5.9 per cent.

The weight of this truss, exclusive of the connecting details, was found to be 4.2% greater than that of the two-hinged arch. This excess in weight is due, mainly, to the influence of the larger reverse

stresses in many of the members. The distribution of weight is as follows: Upper chords, 16.5%; lower chords, 37.2%; diagonals, 22.3%; and verticals, 24.0 per cent. Mr. Jacoby.

Comparing the various classes of members with those of the two-hinged arch, the results are: Upper chords, 9.7% heavier; lower chords, 5.8% heavier; diagonals, 4.8% heavier; and verticals, 1.9% lighter. With the three-hinged arch as a basis of comparison, the upper chord is 34.0% heavier; the lower chord, 9.8% lighter; the diagonals, 16.3% heavier; and the verticals, 7.0% heavier.

The distribution of the material in this combination-type of arch is such that it forms a stiffer structure than the two-hinged arch, its static deflection at the center under full live-load being nearly 10% less.

The writer wishes to express his appreciation of the great care exercised by Messrs. More and Smith in making the computations and graphic constructions involved in these designs, and from which this discussion has been prepared.

In conclusion, a note may be added in the interest of diminishing the labor required to find the stresses. If the stresses in the upper chord due to $H = 1$ and $V = 1$ be computed and laid off on the horizontal lines in Figs. 3 and 6, respectively, experience shows that the rest of the stresses may be determined graphically with a precision that answers fully all the requirements of design, it being understood that a suitable scale is adopted. This procedure will save the tedious computations for the lengths of the lever arms of the remaining truss members.

C. W. Hudson, Assoc. M. Am. Soc. C. E. (by letter).—It is generally fair to assume that the weights of details for trusses of the same type, and not greatly varying spans, are a certain percentage of the weights of the main members, but it is not to be assumed that the same percentage holds between different types of trusses. Mr. Hudson.

In the case of the three-hinged spandrel-braced arch compared with the two-hinged arch, it is evident that the details of the three-hinged arch will be heavier than those of the two-hinged arch by at least the following: First, the weight of the center pin; second, the weight of part of the pin plates bearing on the center pin; third, the weight of additional details, for center top chord and center vertical post, to give proper adjustment for these members for the great range of height of the center pin; and fourth, for the case of the 200-ft. arch spans compared, there was a great saving in details in favor of the two-hinged arch, due to the fact that the arch ring of the three-hinged arch was of greater dimensions, thereby requiring larger gusset plates for connection to web members, also requiring larger and heavier battens and lattice throughout the arch, as the members of the arch other than the ring had to be made wider than necessary in order to take the arch-ring gussets. These heavier battens, lattice, and gussets of the arch

Mr. Hudson. ring could have been avoided by taking a ring of the same size as for the two-hinged arch; but then there would have been a loss of about 1 500 lbs. in the main material of the arch ring, due to the greater value of $\frac{l}{r}$.

From this we see that the fourth saving can be made, either in the details of the entire arch or in the main material of the arch ring. This is due to the fact that there is not such a great difference between the sections of the arch ring and the sections of the other members of the arch for the two-hinged as for the three-hinged arch.